tion of this model, the agreement between theoretical predictions derived from the Marcatili's analysis and experimental data appear satisfactory for most engineering purposes.

ACKNOWLEDGMENT

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Letters

Comment on "Numerical Calculation of Electromagnetic Energy Deposition for a Realistic Model of Man"

KUN-MU CHEN

In the above paper[1], Hagmann *et al.* compared their numerical results on the energy deposition in a model of man with ours [1], [2], and indicated that our results are low compared with their numerical and experimental results. Our low SAR values were obtained because they were based on a simplified model of man which has a reasonable shape but an excessive weight of about 200 kg. We have since published a considerable amount of results based on a more realistic model of man [3], [4] which has a more realistic shape and a weight of about 100 kg. Our numerical results based on the realistic model of man are quite close to the numerical and experimental results of Hagmann *et al.* as shown in Table I.

It is noted that our published results on SAR's [3], [4] are in mW/m^3 and they are induced by an electric field of 1 V/m (max. value), therefore, a factor of 0.00754 should be multiplied to our data to obtain SAR's in W/kg induced by a plane wave with a power density of 1 mW/cm^2.

TABLE I
DISTRIBUTION OF ENERGY DEPOSITION FOR MAN NEAR RESONANCE IN FREE SPACE

Body Part	Experiment, 68 MHz Gandhi et al. [5]	Numerical, 80 MHz Chen et al. [2],[3]	Numerical Hagmann et al. [1]	
			65 MHz	77 MHz
Eye	0.043	0.02	0.0415	0.0427
Neck	6.16 3.13	0.2	0.286	0.318
Heart	0.415	0.32	0.251	0.327
Pelvic Region	0.154	0.17	0.171	0.233
Thigh	0.519	0.62	0.398	0.509
Calf	0.456	0.34	0.543	0.661

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Comments on "Upper Bound Calculations on Capacitance of Microstrip Line using Variational Method and Spectral Domain Approach"

K. SACHSE

The author read with interest the above paper[1] in which an analytical approach based on the Fourier transformation and variational techniques have been employed; the surface potential $V(x)$ of the dielectric sheet in order to find the upper bound of the microstrip line capacitance C^U has been used. Thus this approach complements that of Yamashita *et al* [1], who calculated the lower bound C^L dealing with the charge density $Q(x)$

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[1]K. Araki and Y. Naito, *IEEE Trans. Microwave Theory Tech.*, vol MTT-26, pp. 506–509, July 1978.

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on the surface of the conductor strip; consequently, the margins of error in the variational calculation can be estimated. In the above analysis, the total energy W_e, (10), stored in the system, and by this means the line capacitance C^U, (12), are evaluated applying the Green's first identity and are expressed in terms of the Fourier transform of the surface potential $\tilde{V}(\beta)$. As a supplement of the comments on the reached result in the paper[1], the author's intention is at first to mention that (12), which is rewritten as

$$\frac{C^U}{\epsilon_0}=\frac{1}{2\pi V^2}\int_{-\infty}^{\infty}\tilde{V}^2(\beta)\tilde{F}(\beta)\,d\beta \tag{1}$$

can be evaluated [2] in a simple way using Galerkin's method and Perseval's theorem to the relation between the surface potential and charge distribution, i.e., to the equation

$$\tilde{Q}(\beta)=\tilde{F}(\beta)\tilde{V}(\beta) \tag{2}$$

where $\tilde{F}(\beta)$ is easily obtainable from simultaneous non-homogeneous algebraic equations, settled by using the boundary and continuity conditions.

Moreover, applying the same procedure to the equation

$$\tilde{V}(\beta)=\tilde{F}^{-1}(\beta)Q(\beta) \tag{3}$$

we have strictly obtain the Yamashita's *et al.* [1] variational expression for the lower bound value, namely

$$\frac{\epsilon_0}{C^L}=\frac{1}{2\pi Q^2}\int_{-\infty}^{\infty}\tilde{Q}^2(\beta)\tilde{G}(\beta)\,d\beta \tag{4}$$

where $\tilde{G}(\beta)=\tilde{F}^{-1}(\beta)$ is known as the Green's function. By this means we have evaluated the variational expressions (1) and (4) in the dual form.

Studying the presented numerical results, the comments are as follows. The authors state that "even though a stationary property of the formula (12), it is advisable to choose trial functions close to the true distribution as closely as possible. Moreover, these functions should be select to have simple Fourier transforms suitable for the numerical calculations." However, the functions which have been used in the computations do not satisfy the above convenient properties. The trial function $V(x)=1/|x|$ as well as that given by (18) which is expanded in terms of a negative power series of x, are bad since a pole for $x=0$ and they do not approximate the constant potential of the conducting strip, i.e., when $|x|<a$. By this reason the results presented in the Table I for the upper bound value are poor, namely the error is 48.8 percent for $N=1$ and 6.5 percent for $N=4$, at $2a/h=10^{-1.4}$ (in the text of the paper it has been mentioned the error 2.4 percent for $N=1$, at this point).

On the contrary, the results presented in the Table I for very simple form of the charge approximation, namely for $Q(x)=|x|$ are in very good agreement to the exact values. The function $V(x)$ of the form of (16) is more adequate since the potential is constant for $|x|<a$; unfortunately, the d parameter in (16) is not defined. Though the error of the calculation with the use of the function of (16) is reduced to 9.6 percent, very long computation time is needed, about 5 min per one structure. It is caused by using the trial function of the form of (16) as well as of (18), which are not practical since theirs Fourier transform functions are not elementary ones, given by (17) and (21) to (23). From the author's experience of the problem the exponential functions of the potential distribution can be used with good results, namely, in the space domain

$$V_n(x)=\begin{cases}\dfrac{a}{n}e^{-n}, & |x|\leqslant a\\[2mm] \left(|x|-\dfrac{n-1}{n}a\right)e^{-\frac{n}{a}|x|}, & |x|>a\end{cases} \tag{5}$$

and in the Fourier domain

$$\tilde{V}(\beta)=2\frac{n}{a}e^{-n}\frac{2\frac{n}{a}\beta\cos\beta a-\left[\beta^2+\left(\frac{n}{a}\right)^2\right]\sin\beta a}{\beta\left[\beta^2+\left(\frac{n}{a}\right)^2\right]^2}. \tag{6}$$

Note that the above transforms are suitable for numerical calculations.

In order to calculate the C^U value with the use (12) it is necessary to assume the value of V, i.e., the value of the potential of the conducting strip. What the value has been assumed in case of using of (18)? The condition $V(1)=1$ imposes only a constraint among a_n's. In the paper it has been noted, "The calculated values of the characteristic impedance and guide wavelength are lower bound to the true values." According to the forms of (13) and (14), which can be written as

$$Z=120\pi\epsilon_0/\sqrt{C_0C}$$
$$\lambda=\lambda_0\sqrt{C_0/C} \tag{7}$$

the above conclusion is true for the characteristic impedance only. The authors in the conclusion state also, "Although results are poor for a very narrow microstrip line, they are good for a moderately wide microstrip line," and, "The differences between the lower bounds and the upper bounds are negligibly small for practical purposes." No results in order to prove the validity of this assertion have been presented.

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Reply[2] by Kiyomichi Araki and Yoshiyuki Naito[3]

We are grateful to Dr. Sachse for his critical advise on our paper. As he points out, the Fourier transforms of the surface potential and the charge density, $\tilde{V}(\beta)$ and $\tilde{Q}(\beta)$, respectively, are related to each other by a simple manner

$$\tilde{Q}(\beta)=\tilde{V}(\beta)|\beta|\epsilon_0(1+\epsilon^*\coth|\beta|h).$$

From this relation, the upper and lower bounds of the microstrip line capacitance can be evaluated through the energy integral.

Dr. Sache insists that our trial functions $V(x)=1/|x|$ as well as that given by (18) are inappropriate since a pole exists at $x=0$ and these choices of functions do not approximate the constant potential on the conducting strip, i.e., when $|x|<a$. We believe he misinterpreted our paper. All of our trial functions contain $V(x)=1$ for $|x|<a$. It is stated that $V(x)=1/|x|$ only for $|x|<a$. $V(x)$ in (18) is also for $|x|>a$. The first term of (22) corresponds to the potential distribution $V(x)=1$ on the strip.

The critique also says that the calculated values of the guide wavelength are not lower bound to the true values. In our calculations, the exact values of C_0 are employed, which are obtained by the conformal mapping method. Therefore, the authors believe that our results are all lower bound to the true values.

Finally, the authors would like to add a comment that a fringing field will diminish as the strip width, $2a$ increases and the line capacitance will converge to the value of $\epsilon_0\epsilon^*2a/h$.

[2]Manuscript received May 6, 1980.

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